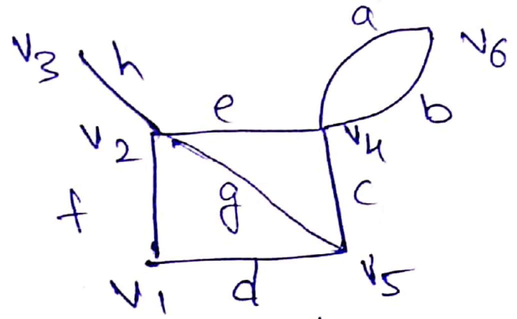
(III) Path matrix (PM)

A PM is defined for a given pair of vertices in a graph, say, (u, v) and is written as $P(u,v)$. Path matrix $P(u,v) = [p_{ij}]$ is defined as—

$$p_{ij} = \begin{cases} 1, & \text{if } j^{th} \text{ edge lies in } i^{th} \text{ path} \\ 0, & \text{otherwise} \end{cases}$$

Rows in $P(u,v)$ correspond to different paths b/w vertices u & v
Col^m^s correspond to the edges in G.

Ex- To find path matrix for v_3 & v_4 in G given as—



There are 3 different paths b/w v_3 & v_4 viz $\{h, e\}, \{h, g, c\}, \{h, f, d, c\}$

Let us call these paths as 1, 2, & 3 respectively. Since there are 8 edges in given graph, ∴ path matrix will be of order 3×8. It is given by—

$$P(v_3, v_4) = \begin{array}{c c} & \begin{array}{cccccccc} a & b & c & d & e & f & g & h \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

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Properties of Path matrix

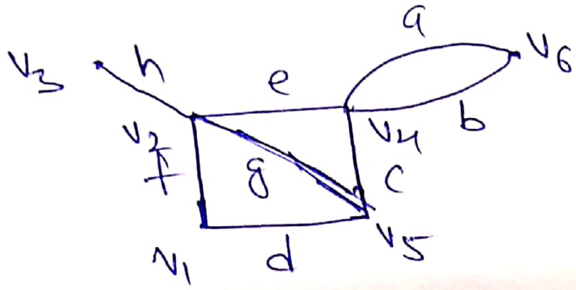
1. A row of all 0's correspond to an edge which does not lie in any path b/w u & v.
2. A row of all 1's correspond to an edge which lies in every path b/w u & v.
3. There can not be row with all 0's

IV. Circuit matrix (CM)

Let G be a graph with 'e' edges and 'q' different circuits. Then a circuit matrix $B = [b_{ij}]$ of G is a $(q \times e)$ matrix defined as—

$$b_{ij} = \begin{cases} 1, & \text{if } i^{th} \text{ circuit includes } j^{th} \text{ edge} \\ 0, & \text{otherwise} \end{cases}$$

Ex: To find C.M. of the graph—



This graph has 4 circuits viz

(1) $\{a, b\}$, (2) $\{c, e, g\}$, (3) $\{d, f, g\}$, (4) $\{c, d, f, e\}$.

The C.M. is—

$$B(G) = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccccccc} a & b & c & d & e & f & g & h \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

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Properties of a C.M

1. If an edge does not belong to any circuit then the corresponding col^n of C.M. contains all zeros

2. A C.M. is capable of representing self loops
3. The number of 1's in a row is equal to the number of edges in the corresponding circuit

4. If graph G is separable (or disconnected) and consists of two components (or blocks) H_1 & H_2 then the C.M $B(G)$ of graph G can be written as—

$$B(G) = \left[\begin{array}{c|c} B(H_1) & 0 \\ \hline 0 & B(H_2) \end{array}\right]$$

where $B(H_1)$ & $B(H_2)$ are the circuit matrices of H_1 & H_2.

Cut-Set matrix

The Cut-Set matrix $C = [C_{ij}]$ of a graph is defined as—

$$C_{ij} = \begin{cases} 1 & \text{, if } i^{th} \text{ cut.set contains } j^{th} \text{ edge} \\ 0 & \text{otherwise} \end{cases}$$

In this matrix, rows correspond to the cut-sets and the col^{n}s to edges of the graph.

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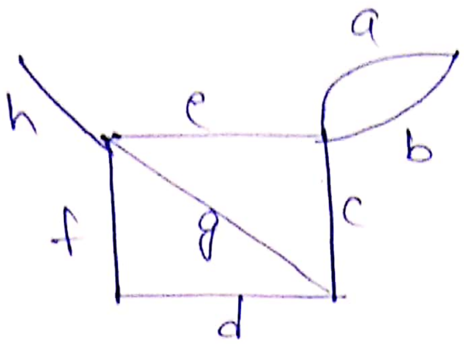
Q. Ex: To find cut-set matrix for the graph.



First we find cut-sets for this graph.
Cut-sets are:

$\{h\}_{(1)}, \{a,b\}_{(2)}, \{e,c\}_{(3)}, \{f,e\}_{(4)}, \{d,c\}_{(5)}, \{f,d\}_{(6)},$
$\{d,g,c\}_{(7)}, \{e,g,d\}_{(8)}$ → 8 cut-sets, 8 edges → 8×8 matrix

∴ Cut-set matrix is given by—

$$C = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{array} \begin{array}{c} \begin{array}{cccccccc} a & b & c & d & e & f & g & h \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

Properties of Cut-Set matrix

1. If a column in a C-S matrix has all zeros then the corresponding edge is a self-loop in the graph and vice-versa.
2. Two edges in a graph are ∥ edges iff corresponding columns in cut-set matrix are identical.

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